Lecturered By Dr. H.K. Yadav.
Deptt. of Mathematics, Marwari College, BSP
B.Sc. class-VI Date - 08-05-2020

Topic: Relations and Functions.
Ex-H.

Q. 1. Check whether the relation R defined in the set $\{1,2,3,4,5,6\}$ as $R = \{(a,b) : b = a+1\}$ is reflexive, symmetric or transitive.

Soln: Let $A = \{1,2,3,4,5,6\}$

A relation R is defined on set A is

$R = \{(a,b) : b = a+1\}$.

$\therefore R = \{(1,2),(2,3),(3,4),(4,5),(5,6)\}$

Now, $6 \in A$ but $(6,6) \notin R$.

Therefore, R is not reflexive.

It can be observed that $(1,2) \in R$ but $(2,1) \notin R$.

Therefore, R is not symmetric.

Now, $(1,2), (2,3) \in R$ but $(1,3) \notin R$.

$\therefore$ R is not transitive.

Hence, R is neither reflexive nor symmetric nor transitive.

Q. 2. Show that the relation R in R defined as $R = \{(a,b) : a \leq b\}$ is reflexive and transitive but not symmetric.

Soln: We have, $R = \{(a,b) : a \leq b\}$. Let $a, b \in R$

Reflexive for any $a \in R$ we have $a \leq a$. So, R is reflexive.

Symmetric: We observe that $(2,3) \in R$ but $(3,2) \notin R$. So, R is not symmetric. For transitivity: $(a,b) \in R$ and $(b,c) \in R \Rightarrow a \leq b$ and $b \leq c \Rightarrow a \leq c \Rightarrow (a,c) \in R$. So, R is transitive. Hence, R is reflexive and transitive but not symmetric.

Proved.

B.Sc - class - XII · Date - 08-05-2020.

Prof.

Q.3. Check whether the relation R in R defined by $R = \{(a,b) : a \leq b^3\}$ is reflexive, symmetric and transitive.

Soln: It is given that $R = \{(a,b) : a \leq b^3\}$.

It is observed that $(\frac{1}{2}, \frac{1}{2}) \in R$ as $\frac{1}{2} < (\frac{1}{2})^3 = \frac{1}{8}$

So, R is not reflexive.

Now, $(1,2) \in R$ [as $1 < 2^3 = 8$]

But $(2,1) \notin R$ [as $2^3 > 1$]

So, R is not symmetric.

We have, $(3, \frac{3}{2}), (\frac{3}{2}, \frac{6}{5}) \in R$ as $3 < (\frac{3}{2})^3$

and $\frac{3}{2} < (\frac{6}{5})^3$

But $(3, \frac{6}{5}) \notin R$ as $3 > (\frac{6}{5})^3$

∴ R is not transitive.

Hence, R is neither reflexive nor symmetric nor transitive.

Q. 4. Show that the relation R in the set $\{1,2,3\}$ given by $R = \{(1,2), (2,1)\}$ is symmetric but neither reflexive nor transitive.

Soln: It is given that $A = \{1,2,3\}$

A relation R on A is defined as $R = \{(1,2), (2,1)\}$

It is seen that $(1,1), (2,2), (3,3) \notin R$.

∴ R is not reflexive.

Now, $(1,2) \in R$ and $(2,1) \in R$. So, R is symmetric.

Now, $(1,2)$ and $(2,1) \in R$ but $(1,1) \notin R$. So, R is not transitive.

Hence, R is symmetric but neither reflexive nor transitive.